# The Real Nash Equilibrium

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The movie Beautiful Mind managed to get Nash's major discovery wrong.

There were a number of historical inaccuracies. The movie portrayed Nash's schizophrenia as a bunch of right-wing fantasies. In reality, they were left-wing fantasies. Nash also spoke with space aliens.

Besides the historical inaccuracies, they got the game theory wrong too. They showed Nash "discovering" Nash Equilibrium in a bar discussion. Except what he described as the Equilibrium is not the Nash equilibrium.

The Nash Equilibrium is a set of strategies where no player has a unilateral incentive to change strategies. This could be a single dominant strategy or a mixed strategy. The player knows what his opponent may do in every circumstance, but the player will not unilaterally change strategies regardless of the other player's moves. In every finite non-zero sum and non-cooperative game, there is at least one circumstance where there is a Nash Equilibrium.

So here's the scene on YouTube: Russell Crowe is sitting in a bar and he suddenly shouts
"Adam Smith needs revision!"
Nope.
"Adam Smith said the best result comes from everyone in the group doing what's best for himself, right? Adam Smith was wrong!"
Surreal.

So in the movie, Nash observes a blonde and four brunettes enter a bar. He and his three friends would like to ask them out. All the men prefer the blonde over the brunettes. But, if they ask out the blonde first and get rejected, they cannot score with any of the brunettes.

Movie Nash:

> *"If everyone competes for the blonde, we block each other and no one gets her. So then we all go for her friends. But they give us the cold shoulder, because no one likes to be second choice. Again, no winner. But what if none of us go for the blonde. We don't get in each other's way, we don't insult the other girls. That's the only way we win."*

The Movie Nash says that all three men should ask out only the brunettes and ignore the blonde. That way they will maximize their expected utility of scoring and minimize their losses. They will all benefit the most if they cooperate! This is why Adam Smith is wrong.

> *"Best result comes from everyone in the group doing what's best for himself. Incomplete. Incomplete. Because the best result will come when everyone in the group doing what's best for himself and the group."*

Well no, by definition, John Nash explored non-cooperative games. Individual strategies do what is best for the individual, not the group.

Movie Nash described the only result in the mating game that is *not a Nash Equilibrium*. Real-World Nash actually discovered that equilibria are not always optimal. The Prisoner's Dilemmais the classic example of a Nash Equilibrium where both players refuse to cooperate and gain far less than mutual cooperation. The Prisoner's Dilemma occurs because neither player has an incentive to unilaterally change strategies to cooperation.

So let's look at this bar game. There are 4 Brunettes and 1 blonde. The men's preferences are Blonde > Brunette > Nothing. Let's attach values to this. Scoring with the blonde is worth 10, a Brunette is 5, and Nothing is 0.

True enough, if all four men swarm the blonde, they each get 0. If all four men each take a brunette, they get a payoff of 5 (The blonde presumably gets 0, but the females don't count in the game).

However, here is the problem. Every male has an unilateral incentive to "defect" from brunette cooperation. If he knows every other male will ask out the brunettes, this leaves the blonde available. The total utility of the blonde (10) is greater than the brunette (5), and without competition he has a much higher probability scoring with the blonde.

Got that? Every male has a unilateral incentive to defect from a pure brunette cooperation strategy.

There are many Nash Equilibria in this game. Some of mixed strategies (a mixed probability of selecting a brunette or blonde – Say go for the Brunette 60% of the time and the Blonde 40% of the time) and some are pure "blonde" strategies. The only thing that is not a Nash Equilibrium is a pure brunette strategy.

John Nash did not overturn 150 years of economic theory as he claimed in the movie. The reason why markets work is because of price signals, which are lacking in the mating game. The work of Kenneth Arrow and many others demonstrated this with General Equilibrium Theory. Adam Smith was right because he described how self-interest produces cooperation under certain circumstances.

Nash Equilibrium explains what happens in non-cooperative games in non-market circumstances. Nash's bargaining solution shows that mating games are inefficient and tend towards bad results because women do not price themselves on a market. This leads to less than optimal results.

An efficient mating game would use a market signal supply and demand. The blonde is in short supply but in high demand, so she would sell for a much higher price, while the brunettes lower their price to make themselves more attractive. The price levels out at a mutually beneficial equilibrium. This enables market cooperation even though everyone acts in self-interest.

Without a market, the men would compete for the blonde and increase the number of losers. In really bad non-cooperative mating games, there are a Prisoner's Dilemma-like results.

It's not that Hollywood gets the history wrong. They got everything wrong once again. They took a brilliant mathematical theory and stated that the exact opposite as true. Because, I guess, saying "cooperation" *sounds nice*. Nash proved that people will not cooperate in some games.